The Calculation of Costs for Delivery Routes

von

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1. Introduction

In this paper delivery routes for local delivery are examined which start with a loaded delivery vehicle in a depot, then customers are found consecutively and the ordered goods are delivered to them. After its last delivery to a customer the delivery vehicle returns to its depot. Since the service provision for delivery routes represents network efficiency, the question to be answered is in which way the total costs of a delivery route should be distributed to the individual customers. A billing procedure has to be reasonable and it also has to appear to be fair. This is also necessary since logistic partners have to make their account costing accessible for outsourcing discussions and they have to be created in a manner that is understandable for the customer (Christopher 2010). Furthermore with this approach the advantage of gaining a new customer on a route is determined in the cost calculation and implemented in dynamic pricing strategies for bottlenecks. So far this question has not been dealt with in literature for vehicle routing (see Golden et al. 2008, Toth et al. 1987 and Noureddine 2009).

Applications for the cost distribution discussed in this paper consist of, for example the cargo transport delivery and for the supply of grocery stores from large grocery chain’s regional warehouses. For parcel service delivery routes, which can contain up to 100 stops in the urban areas, the distribution of route costs to customers is not important because of the price model of standard packages.

2. The Concept of the Methode

On the basis of the considerations a marginal cost criterion is offered. For this the costs of a delivery route should be considered and the cases should be distinguished for how the delivery costs can be reduced, when a customer i is excluded from the delivery route. This difference in costs then represents the differential costs for the delivery to a customer i.

These differential costs are used in order to disperse the costs of a delivery route in the basic costs and the additional costs. While the basic costs are proportionally distributed to the individual customers of the delivery route according to the transport service criteria, the additional costs are to be deducted proportionally from the differential costs. This rough outline of the concept will now be refined and precisely defined in what follows.
In order to state the addressed problem more precisely, we will assume a model in which the depot together with $n$ different customers are present in a road network. The depot is numbered by 0, and the customers are denoted by $i$, $i = 1, \ldots, n$. In general for every two customers, there are different routes between them in the road network. Because of the high importance of speed in logistics systems and to realize the delivery vehicles (in the following: trucks) highest possible delivery services on a daily basis, we generally consider the fastest route. With the shortest route method the distances between every two customers $i$ and $k$, and between depot 0 and every customer $i$ on the fastest route can be determined. The length of the fastest route is measured in kilometers. For the tolls, on the fastest route the covered kilometers between customer $i$ and customer $k$ on the expressways are charged separately. For every customer $i$ a load quantity $L_i$ is stored on every truck. This quantity can either be measured in units of weight (kilograms), units of volume (cubic m) or in pallet spaces.

The calculation for individual customer’s creditable expenses is performed in three steps:

- First the total costs of the route are determined as full costs.
- Then the total costs are split into the two components basic costs and additional costs. The additional costs are determined through the marginal cost criterion, which is described above and are deducted from the total costs. The difference represents the basic costs.
- Finally the basic costs and additional costs are broken down to the individual customers.

3. The Calculation of the Total Costs

The total costs for route $T$ are calculated in the following way (cf. Rushton and Baker 2010, chapter 27): The total costs $TC$ are determined by fixed elements of costs and by variable components. The annual fixed costs of the truck are broken down into a daily rate and hourly rate, and related to the travel time of a route. The duration of route $T$ is measured from the beginning of the route at the depot until the end of the route at the depot in hours. This duration is denoted as working time and multiplied by the hourly rate, which also includes the costs for the driver.

Also the distance-dependent variable costs are added to fixed costs determined by duration, which include the maintenance costs, the diesel fuel consumption and the truck’s expressway tolls. The maintenance costs and the costs for the diesel fuel are summarized as the consumption rate per kilometer. The second cost component is distance-dependent and when it is included in the truck toll it looks like the following:
Driven kilometers* Consumption rate +
Driven expressway kilometers* Truck toll rate

As a result the total costs TC are dependent on a time factor and distance factor:

\[ TC = \text{Working time} \times \text{Hourly rate} + \text{Driven kilometers} \times \text{Consumption rate} + \]
\[ \text{Driven expressway kilometers} \times \text{Truck toll rate} \]

If the truck cannot be used after returning from the route at the depot for the rest of the day, because the remaining time is too short, a daily rate applies to the time-dependent cost component.

4. The Calculation of the Additional Costs

Now in a second step the additional costs are determined. For this purpose the tax credit method needs to be modified, when an individual customer i (or a group of customers) requires a major detour in order to be supplied. This case occurs if there is a shorter alternative route in the road network that can be driven without supplying customer i. But this is not possible in all cases. If for example customers are located along a through street, then no alternative routes are available (cf. figure 1, case D and the discussion below).

When detours for delivery to customers are necessary, each detour induces costs in these three categories

- driven kilometers (detour distance),
- toll costs for driven expressway kilometers (detour toll) and
- time spent (detour time)

which are charged separately to these customers and customer groups.

4.1 The Calculation of the Detour Distance

We assume that the delivery route T was determined by the optimization methods of the Travelling Salesman. These methods ensure that the truck in the given road network is on a route with a minimum travel time on the fastest routes. Additionally we consider a route T(i), which

1. results from route T by removing customer i, and then
2. is to be determined by the optimization methods of the Travelling Salesman, since route \( T(i) \) does not emerge from merely leapfrogging customer \( i \) from the route \( T \) (see Surhone et al. 2009).

In order to determine customer \( i \)’s detour distance \( U_i \) for a delivery route \( T \), we assume a distance for route \( T \) which is determined as the sum of the driven kilometer distance, covered for the route and denoted as \( D(T) \). The distance of the route \( T(i) \) is denoted as \( D(T,i) \).

The **detour distance** \( U_i \) then results from the difference between the route’s distances \( T \) and \( T(i) \):

\[
U_i = D(T) - D(T,i).
\]

The detour distances depend on the topology of the delivery routes. We are talking about five different cases, which are shown in figure 1. Part A) describes a route which starts in the depot and ends, and consecutively visits 6 customers \( C_1,\ldots,C_6 \). The route looks like an almost regular convex figure in the plane. As an example for the detour distance for individual customers, the customer \( C_2 \) is singled out here. The dashed line illustrates the road on the route \( D(T,2) \) which would have been taken, if the customer \( C_2 \) is excluded from the route. Information for this case is given in the example below.

While for case A) the data analysis is necessary in order to decide which detours need to be charged separately, it can directly be recognized in cases B) and C), which detours the delivery requires for a customer \( C_2 \) and a customer group \( C_2, C_2A, C_2B \) that lives farther away. For this, larger than average variations of the detour distance \( U_2 \) are incurred by removing customer \( C_2 \) and by removing the customer group \( C_2, C_2A, C_2B \).

Case D) illustrates a linearly ordered delivery route of four customers along a street. In this case by removing one of the customers \( C_1, C_2 \) and \( C_3 \) does not shorten the route, since there is no alternative route available. Thus the detour distances for the customers \( C_1, C_2 \) and \( C_3 \) are equal to zero. For the customer \( C_4 \) the detour route is twice as long as the distance from \( C_3 \) to \( C_4 \). Since it is arbitrary that a positive detour route appears for the last customer, but not for the preceding customer, the detour distances concept cannot be applied in the case for a linear arrangement of a route as criteria for the cost deduction.
It can be similar for case E), where on the way there as well as on the way back a linear structure is driven, but which lies on different routes. In this case the detour costs for the customers C1, C2, C3, C6 and C7 are equal to zero. Detour costs for the customer C4 can also be equal to zero, when the road from C3 to C5 leads only to one orthogonal road network of the city-block metric. Therefore also the application of the concept of detour routes is problematic.
What follows from this discussion, is that a useful application of the concept of the detour distances is only possible for such routes, for which in the case of a removal of a customer an alternative route is available as the fastest route.

For the series of the determined detour distances $U_i$, $i = 1,...,n$, the average $\mu$ and the standard deviation $\sigma$ are calculated. In order to exclude complicated special cases, $n \geq 4$ is assumed. In the following only detours $U_i$ are supposed to be considered, which are “large”, while small detours in the network continue and are not charged separately. In the empirical data analysis often such values are classified as “large”, which are above the limit $\mu+\sigma$. This is however not supposed to happen here, since the large values of $U_i$ are entered in the average $\mu$ and through this raise the value $\mu$. A fair charge for the detour costs requires much more to account for all the detour costs, which are all above the effective minimum. It is therefore suggested that the value $\mu - \sigma$ is set as the limit. To exclude complicated special cases, only series with $\sigma < \mu$ are considered. To neglect small values of the detours can justified with the argument that difficulties arise when small values should be measured exactly.

Consequently a customer $i$ is charged for a detour, when the detour distance is $U_i > \mu - \sigma$. We thus set to

$$
\hat{U}_i = 0, \quad \text{when } U_i \leq \mu - \sigma \text{ and }
$$

$$
\hat{U}_i = U_i - \mu + \sigma \quad \text{if not.}
$$

The choice for $\mu - \sigma$ as a limit is illustrated by the following example: We consider a route $T$ from illustration 1, case A) and the depot locations and customers $C1,...,C6$ are placed underneath with the Euclidean coordinates from table 1.

<table>
<thead>
<tr>
<th>Node No.</th>
<th>$x$-coordinates</th>
<th>$y$-coordinates</th>
<th>Detour distance $U_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>77</td>
<td>57</td>
<td>$d_{01} + d_{12} - d_{02} = 22.2 + 22.3 - 43.2 = 1.3$</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
<td>75</td>
<td>$d_{12} + d_{23} - d_{13} = 22.3 + 18.9 - 36.0 = 5.2$</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
<td>85</td>
<td>$d_{23} + d_{34} - d_{24} = 18.9 + 20.6 - 37.6 = 1.9$</td>
</tr>
<tr>
<td>3</td>
<td>126</td>
<td>75</td>
<td>$d_{34} + d_{45} - d_{35} = 20.6 + 19.2 - 34.2 = 5.6$</td>
</tr>
<tr>
<td>4</td>
<td>134</td>
<td>56</td>
<td>$d_{45} + d_{56} - d_{46} = 19.2 + 27.0 - 42.1 = 4.1$</td>
</tr>
<tr>
<td>5</td>
<td>122</td>
<td>41</td>
<td>$d_{56} + d_{60} - d_{50} = 27.0 + 24.7 - 47.8 = 3.9$</td>
</tr>
</tbody>
</table>

Table 1. data for case A) in Illustration 1
To simplify we will use Euclidean distances $d_{ij}$ between nodes i and j, instead of distance measurements in a road network. Because of the convex shape of the route $T$ or $T(i)$ optimized by methods of the Travelling Salesman it is determined as a round trip on the convex node cover (Surhone et al. 2009). We will also assume that the fastest route is on the shortest route. Through this we obtain the series of the detour distances $U_i$ from table 1, which exhibits an average $\mu = 3.7$ and a standard deviation $\sigma = 1.6$. As a threshold value $\mu - \sigma$ we obtain the value 2.1. After which the detour distances for the customers C2, C4, C5 and C6 are charged separately. Whereas when the choice is made for $\mu + \sigma$ only the detour for C4 is taken into account separately.

While it is being suggested that the limit for the detour charge is to be set by $\mu - \sigma$, also other values can be used for this, which lie between the minimum detour value $\text{Min}(U_i)$ and $\mu - \sigma$. The value to be used in practice should be determined by an empirical study.

If not just one customer, but a whole group of customers is far way from the remaining route $T(i)$, then the detour costs can also apply to the group of customers. If for example three customers i, h, m can only be reached by a detour, but they are all close to each other, then the detour distances should be calculated as $U(i,h,m) = D(T) - D(T,i,h,m)$ (refer to figure 1, case D for the customers C2, C2A, C2B). The size $D(T,i,h,m)$ is then the distance according to the criteria for the shortest travel time with the optimized route to customers by the methods of the Travelling Salesman. The customers i,h,m are removed from the route $T$.

**Discussion:** The approach used here for determining the sizes $\hat{U}_i$ at first appears as very schematic. Instead a piecewise linear function $f(x) = 0$ is used for $x \leq \mu - \sigma$ und $f(x) = x - \mu + \sigma$ für $x > \mu - \sigma$ to deduce the sizes $\hat{U}_i$ from the sizes $U_i$. For this a function can be used, which starting from the limit $\mu - \sigma$ becomes increasingly steeper, in order to charge more for larger detour costs, like a quadratic process. However the application of such functions makes the tax credit method less transparent, which can make the negotiation processes in the Supply Chain more difficult.

4.2 The Calculation of the Detour Toll

In order to charge individual customers for toll costs, for both routes $T$ and $T(i)$ especially the road network of the expressways needs to be considered:

- $\text{DE}(T) =$ driven kilometers on the expressway in route $T$
- $\text{DE}(T,i) =$ driven kilometers on the expressway in route $T(i)$
The difference of the driven kilometers on the expressway is \( \text{UE}_i = \text{DE}(T) - \text{DE}(T, i) \). For the sizes of \( \text{UE}_i \) like in the method above, the average \( \mu \text{E} \) and standard deviation \( \sigma \text{E} \) need to be determined. In the following only detours \( \text{UE}_i \) should be considered, which are larger than average sizes and deviating more than \( \mu \text{E} - \sigma \text{E} \) upward. We set

\[
\hat{\text{UE}}_i = 0, \quad \text{when } \text{UE}_i \leq \mu \text{E} - \sigma \text{E} \quad \text{and} \\
\hat{\text{UE}}_i = \text{UE}_i - \mu \text{E} + \sigma \text{E}, \quad \text{if not}
\]

### 4.1 The Calculation of the Detour Time

With the assumption of an average speed \( v \) in kilometers per hour and the route determined in kilometers, the travel time can be derived in hours from the formula

\[
\text{Travel time} = \frac{\text{route}}{v}
\]

For this the particular travel time expenditures for the customers farther away need to be considered for the size \( \hat{\text{UE}} / v \), whereby for the combined customer groups there is respectively only one representative quantity. Standing times by the customers are considered to be approximately equal and are not accounted for separately.

### 5. The Determination of the Basic Costs and the Costs to Charge Customers

For the determination of the basic costs \( \text{BC} \), the three elements of costs from the total costs \( \text{TC} \) for the detour traveling need to be deducted. From this the size \( \text{BC} \) emerges representing the basic costs, which are determined by the following formula:

\[
\text{BC} = (\text{Working time} - \sum \hat{\text{UE}} / v) \times \text{Hourly rate} + (\text{D}(T) - \sum \hat{\text{UE}}) \times \text{Consumption rate} + (\text{DE}(T) - \sum \hat{\text{UE}}_i) \times \text{Truck toll rate}
\]

With this construction the time slices and kilometer slices of the customers are deducted from the total costs, which represent a „detour“ from the delivery route. Now the distribution of the basic costs to the individual customers in the route is carried out. When one requires a billing procedure to be reasonable and fair for the customers, you can proceed as follows.
To meet the criteria to be fair and reasonable is can be done when the individual customers’ charged costs also rise by:

- the transported amount $L_i$, as well as with
- the distance $d_{0i}$ of the customer i from the depot on the fastest route.

That is why also by default thereof others, the easily arrangeable criteria for the customers should be used for determining the basic costs for creditable costs proportional to transport services, that is $L_i*d_{0i}$ should be determined for customer i. That is why for all the customers the transport services $TS_i = L_i*d_{0i}$ need to be determined and should then be passed on with attributable percentages to the basic costs. For customer i the creditable percentage $p_i$ for the sum of the individual transport services, $\sum TS_i$, is then determined by

$$p_i = \frac{TS_i}{\sum TS_i}.$$ 

The omitted basic costs $BC_i$ for customer i should then be set as a part of the basic costs:

$$BC_i = p_i*BC.$$ 

So that the accounting is consistent the determined costs $BC_i$ may not exceed the transport costs for a delivery to customer i with a single trip from the depot to customer i on the fastest route.

The three determined elements of costs above for the detour traveling are then to be assigned to the individual customers i and together with the percentage of the basic costs they summarize a cost item $C_i$. We then obtain the formula for the costs $C_i$, which will be charged to customer i:

$$C_i = BC_i + \left(\frac{\bar{U}_i}{v}\right) \text{Hourly rate} + \bar{U}_i \text{Consumption rate} + \bar{U}_E \text{Truck toll rate}.$$ 

When the detours are small or equal zero, as in the cases D) and E) in figure 1, the total costs are equal to the basic costs and the customers are charged accordingly to their share $p_i$ of the basic costs.

6. **Conclusion**

The paper presents a method to distribute the total cost of a delivery tour to the individual customers. For the implementation of the method in the daily operations a software tool has to
be developed that makes use of the geo data of the customers and calculates the detours with fast heuristic methods of the traveling salesman approach.

7. **Literatur**